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# ON THE DETERMINATION OF THE CONTROL PARAMETERS OF THE OPTIMAL CAN-ORDER POLICY

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**Abstract:**

This paper considers the well-known class of can-order policies. This type of coordinated replenishment policies accounts for a joint set-up cost structure, where a major set-up cost is incurred for any order and an individual minor set-up cost is charged for each item in the replenishment. Recent comparative studies have pointed out that the performance of the optimal can-order policy is poor, compared to other coordinated replenishment strategies, when the major set-up cost is high. This paper shows that it is the traditional method to calculate the optimal can-order parameters which performs bad in such situations and not the policy itself. Attention is focused to a subclass of can-order policies, which is close to the optimal can-order policy for high major set-up costs. A solution procedure is developed to calculate the optimal control parameters of this policy. It is shown that a properly chosen combination of the solution procedures to calculate can-order parameters leads to a can-order strategy which performs as well as other coordinated replenishment policies.

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## 1. Introduction

The main part of inventory management literature is focused on independent replenishments of single items, whereas joint replenishments are common practice in real-life procurement processes. The coordination of replenishment orders may lead to considerable cost savings as a result of reduced ordering costs, reduced freight rates, reduced handling costs, quantity discounts or improvement of the implementation of stock control. A realistic way to model the cost effectiveness of coordination is by the joint set-up cost structure, where a major set-up cost is incurred for any order and an individual minor set-up cost is incurred for each item in the replenishment. So, the major set-up cost, associated with each order, is shared when two or more items are jointly replenished.

The inventory management literature on joint replenishment systems has mainly been focused on this cost structure. Recent reviews are given by Aksoy and Erenguc (1988) and Goyal and Satir (1989). For the case of stochastic demand, the optimal policy for the joint replenishment problem is unknown (except for the special case of two items with Poisson demands (see Ignall, 1969)). Therefore, attention has been focused on special ordering policies, which are on one hand close to the (unknown) optimal policy and on the other hand are theoretically analyzable and easy to implement.

Most extensively studied is the class of can-order policies, which are characterized by a set of three parameters $(S_i,c_i,s_i)$ for each item i. Inventory levels are continuously monitored under this type of control. Item i will trigger a replenishment order whenever its inventory position is at or below the 'must-order point' $s_i$. At the same time, any item j with an inventory position at or below its 'can-order point' $c_j$ is included in the joint replenishment. The inventory position of every item j in the order is raised up to its 'order-up-to-level' $S_j$. Silver (1974, 1981), Thompstone and Silver (1975) and Federgruen, Groenevelt and Tijms (1984) developed algorithms to find approximations of the parameters of the optimal can-order policy in case of (compound) Poisson demands.

Another coordinated continuous review system is provided by the class of QS-policies, which use a group reorder point to trigger an order. Under this policy, the inventory position of all items j is raised up to the order-up-to-level $S_j$ whenever the combined inventory position of all the items drops to or below the group reorder point. Under unit demand sizes, the combined order quantity is Q and the group reorder point is reached whenever the total demand since the last order reaches Q. In case of Poisson demands, Pantumsinchai (1992) developed an algorithm to determine the parameters (Q and $S_i$ for each item i) of the optimal strategy within the class of QS-policies.

In the literature there have also been suggested several coordinated periodic review policies which usually are generalisations of periodic single-item policies with synchronized review intervals. An example of such a multi-item system is a RS policy (determined by the parameters $(R_i,S_i)$ for every item i), where the inventory position of item i is ordered up to $S_i$ every $R_i$ periods. To achieve coordination, the review-intervals $R_i$ are chosen as multiples $k_i$ of some basic period. (See e.g. Chakravarty (1986) and Atkins and Iyogun (1988)). Other coordinated periodic policies are suggested by IBM (1971), Naddor (1975), Carlson and Miltenburg (1988), Chakravarty and Martin (1988) and Sivazlian and Wei (1990).

Recently, Atkins and Iyogun (1988) and Pantumsinchai (1992) compared the performance of different coordinated replenishment policies under Poisson demands. They concluded from their empirical results that the optimal QS and RS strategy outperform the 'optimal' can-order strategy quite frequently. The performance of RS and QS policies compared to the can-order policy improves as the major set-up cost (relative to the average minor set-up cost) increases and reaches improvements up to 20%. In these comparative studies, the can-order parameters were calculated by the method of Federgruen et al. (1984). Section 2 shows that the bad performance of the can-order policy is due to the decomposition assumption which is used by Federgruen et al. As a consequence, it is the *method* to calculate the can-order parameters which performs bad in situations with high major set-up costs, but not the can-order *policy* itself. For high set-up cost ratios (i.e. the ratio of the major set-up cost and the average minor set-up cost), attention is restricted to the subclass of can-order policies with $c_i = S_i - 1$ for all items i. Under this policy all items are jointly reordered as soon as one item reaches its must-order point. Section 3 analyzes this policy and develops a solution procedure to determine the set of parameters $(S_i,s_i)$ for each item i. In Section 4, the performance is compared with the performance of the can-order strategy obtained by the traditional algorithm as well as the optimal QS and RS strategy. Finally, the major conclusions are summarized in Section 5.

## 2. Evaluation of traditional approach to determine can-order parameters

Consider a family of N items with demands generated by independent Poisson processes with rate $\lambda_i$ for item i. Unsatisfied demands are completely backlogged. The replenishment lead time of an order is deterministic and equals L periods. The major set-up cost, associated with any order, is denoted by A, and the minor set-up cost, for each item i included in the replenishment, is $a_i$. Let $\bar{a}$ be the average minor set-up cost, then the set-up cost ratio is defined by $A/\bar{a}$. Holding costs are charged at a rate $h_i$ per period on every unit of item i on stock. The management requires that a given fraction $\beta$ of demand has to be satisfied directly from stock on hand. The criterion is to minimize the sum of the long run average holding and ordering cost subject to the service constraint.

Although the control mechanism of the can-order policy is very simple, it is difficult to determine the optimal control parameters ($S_i$,$c_i$,$s_i$, i=1,..,N). The main complication is caused by the interaction between items. When an order is triggered by item i, because its inventory position falls to $s_i$, this represents a *special replenishment opportunity* to order at reduced set-up costs for all the other items. Silver (1974) suggested to decompose the N-item problem in N single-item problems by assuming that special replenishment opportunities for item j (the trigger moments of all the other items) occur according to a Poisson process with rate $\mu_j$ which is independent of the demand process of item j. Let $\xi_i$ denote the expected number of replenishments per unit time that is triggered by item i, then $\mu_j = \sum_{i \neq j} \xi_i$ . This idea was used in the papers by Silver (1974, 1981), Thompstone and Silver (1975) and Federgruen et al. (1984). They developed solution procedures to find the optimal parameters $S_i$,$c_i$,$s_i$ for item i in the resulting single-item problem with special replenishment opportunities occurring at a given rate $\mu_i$. The actual rates $\mu_i$ of special replenishment opportunities are calculated by an iterative procedure.

The procedure of Federgruen et al., which uses a specialized policy iteration algorithm, gives exact cost expressions when the decomposition assumption holds. Silver (1974) already noted that the special replenishment opportunity model tends to overestimate the real cost and to underestimate the real service. These findings were confirmed by our own simulation results. The extent of overestimation of the real cost increases as the set-up cost ratio increases. The conclusions in the comparative studies of Atkins and Iyogun (1988) and Pantumsinchai (1992) are based on the cost which are computed from the *model* of Federgruen et al. In our opinion, it would be better to use in these comparisons the real (simulated) cost of the can-order strategy, which is suggested by the model. In Table 1, the simulated cost is compared with the model cost for the examples in Table 5 of Atkins and Iyogun (1988). It turns out that the percentage cost error may be significant.

**Table 1:** Comparison of model cost and simulated cost

| example | model cost | simulated cost | % cost error |
|---|---|---|---|
| 1 | 1929 | 1626 | 18.63 |
| 2 | 1991 | 1676 | 18.79 |
| 3 | 2043 | 1727 | 18.30 |
| 4 | 1869 | 1610 | 16.09 |
| 5 | 1504 | 1263 | 19.08 |

Note: Input-data are identical to Table 5 in Atkins and Iyogun (1988).
% cost error = 100 · (model cost-simulated cost)/simulated cost.

When the set-up cost ratio is zero, then the optimal can-order policy will be an independent policy with $c_i = s_i$ for all items i. On the other hand, when the set-up cost ratio is infinite (because the minor set-up cost is negligible for each item), then the optimal policy has $c_i = S_i-1$ for all items, which implies that all items are jointly replenished as soon as an item triggers an order. (Since $c_i = S_i-1$, an item is not ordered if there has been no demand for it after the preceding order). The above mentioned two special policies can be considered as extreme policies within the class of possible can-order policies.

One may imagine that the optimal can-order policy will tend to a (S,S-1,s) policy for high set-up cost ratios. Since all items are ordered simultaneously under a (S,S-1,s) policy, the control parameters ($S_i$,$s_i$, i=1,..,N) have to be chosen such that the residual stock (i.e. the stock above the must-order point when an order is triggered) will be close to zero for every item. This implies that during a cycle between two trigger moments the probability of a special replenishment opportunity will be rather low in the beginning of the cycle and high at the end. This contradicts with the approximate assumption of Poisson arrivals of special replenishment opportunities, which is made by Silver, Federgruen and others. Numerical examples point out that the misspecification in the cost of a reasonable (S,S-1,s) strategy is very high if the method of Federgruen or Silver is used. In fact, their models will hardly suggest a strategy of (S,S-1,s)-type because the cost of such a strategy is overestimated even more than can-order strategies with other parameter settings.

Hence, we conclude that the traditional approach to determine the can-order parameters leads to bad results for high set-up cost ratios because in this situation the optimal solution does not satisfy the assumption of Poisson arrivals of special replenishment opportunities. In the next section, an alternative solution method is proposed for these cases. This method determines the parameters of a (S,S-1,s) policy, which is, in general, close to the optimal can-order policy in situations with high set-up cost ratios.

## 3. Determination of the parameters of the optimal (S,S-1,s) policy

This section is divided in three parts. In the first part, a cost expression is derived for a given (S,S-1,s) strategy. The second part develops a method to find the must-order point $s_i$ $(i=1,..,N)$ given a vector $\mathbf{\Delta}:=(\Delta_1,..,\Delta_N)=(S_1\text{-}s_1,..,S_N\text{-}s_N)$. Finally, the results of the first and the second part are used in the third part, which presents a heuristic algorithm to determine the optimal parameters of a (S,S-1,s) policy.

### *3.1. Cost expression for a given (S,S-1,s) strategy*

Note that the inventory position of each item i equals $S_i$ at the beginning of an order cycle, which ends as soon as any item reaches its must-order point. The stochastic process, which describes the changes in the vector of the inventory positions just before an order, is a discrete-time Markov chain with a finite state space.

For a given (S,S-1,s) strategy, define:

$C$ : long run average cost per unit time;
$p_i^0$ : probability that no demand arrives for item i during an order cycle;
$\eta_i$ : expected holding cost of item i during an order cycle;
$\tau$ : expected length of an order cycle.

Then, from the theory of regenerative processes, it follows that

$$C = \frac{A + \sum_{i=1}^{N} \left\{ (1 - p_i^0) a_i + \eta_i \right\}}{\tau} . \tag{1}$$

Suppose an order cycle starts at time 0. To analyze the expected (order) cycle time, define the following stochastic variables:

$T_i$ : time until the cumulative demand for item i reaches the level $S_i$-$s_i$;

$T$ : time until *any* item triggers an order.

Note that item i will trigger an order as soon the total demand for item i from time 0 onwards equals $S_i$-$s_i$. Because demands for individual items are generated according to independent Poisson processes, it follows that $T_i$ is Erlang-distributed with parameters $\lambda_i$ and $S_i$-$s_i$. Denote the corresponding probability density function and the distribution function by $f_i(t)$ and $F_i(t)$ respectively. Noting that $T = \min_i T_i$ it follows that the distribution function and the density function of T, denoted by F(t) and f(t) respectively, are given by

$$F(t) = 1 - \prod_{i=1}^{N} (1 - F_i(t)) , \tag{2}$$

and,

$$f(t) = \sum_{i=1}^{N} f_i(t) \prod_{j \neq i}^{N} (1 - F_j(t)) . \tag{3}$$

The expected length of an order cycle is then given by

$$\tau = \int_{t=0}^{\infty} \{1 - F(t)\} dt = \int_{t=0}^{\infty} \{\prod_{i=1}^{N} (1 - F_i(t))\} dt . \tag{4}$$

This integral can be approximated arbitrarily close by numerical integration.

Define:

$\Phi_i(k)$ : the probability that at time T the residual stock of item i equals k;

$H_i(x,y,t)$ : expected total holding cost for item i during an order cycle of t periods given that the inventory on hand equals x at the beginning and equals y at the end of the cycle.

The probability mass function of the residual stock of item i ($i=1,..,N$), which turns out to be an important factor, is determined in Appendix 1.

Consider the expected holding cost per order cycle in case the lead time is negligible. Then, the inventory on hand of item i decreases from $S_i$ to $s_i+k$ ($k=0,..,S_i-s_i$) with probability $\Phi_i(k)$ during an order cycle. If the order cycle time is t periods, the expected holding cost during that cycle equals $H_i(S_i,s_i+k,t)$. A general expression for $H_i(x,y,t)$ is derived in Appendix 2.

The problem of determining the expected holding cost during a cycle is complicated when there is a positive lead time L because the inventory position and the inventory on hand differ during a lead time L after an order. A standard convention to handle positive lead times, which is also used by Federgruen et al., is to shift the holding cost in [L,T+L] back to the interval [0,T]. Because the demand for item i during the lead time L is generated by an independent Poisson process with rate $\lambda_i L$, the inventory on hand at time L equals $S_i$-j with probability $\frac{(\lambda_i L)^j}{j!}e^{-\lambda_i L}$.

Now, it is easily seen that

$$\eta_i = \sum_{j=0}^{\infty}\frac{(\lambda_i L)^j}{j!}e^{-\lambda_i L}\sum_{k=0}^{S_i-s_i}\Phi_i(k)\int_{t=0}^{\infty}H_i(S_i-j,s_i+k-j,t)f(t)\,dt\,. \qquad (5)$$

Using formula (a1), (a3) and (a4), equation (5) can be approximated arbitrarily close by numerical integration.

Finally, the probability $p_i^0$ is equal to $\Phi_i(S_i-s_i)$. This completes the derivation of the elements of cost formula (1).

### *3.2. Determination of the must-order points*

This subsection investigates the determination of the must order points given a vector $\Delta=(\Delta_1,..,\Delta_N)=(S_1-s_1,..,S_N-s_N)$. The problem is to find the lowest value of $s_i$ ($i=1,..,N$) such that a given fraction of demand, $\beta$, is satisfied directly from stock on hand.

Define, for a given (S,S-1,s) strategy, for item i:

$\mathscr{F}_i$ : long run fraction of demand satisfied directly from stock on hand;

$ES_i$ : expected number of shortages during an order cycle;

$EQ_i$ : expected order quantity per order cycle.

From the theory of regenerative processes, it follows that

$$\mathcal{F}_i = 1 - \frac{ES_i}{EQ_i} . \tag{6}$$

Recall that $\Phi_i(k)$ is the probability of having a residual stock of k units for item i at time T and that the demand for item i during the lead time is generated by a Poisson process with rate $\lambda_i L$. Then it easily follows that

$$ES_i = \sum_{k=0}^{\Delta_i} \Phi_i(k) \sum_{j=s_i+k}^{\infty} (j-s_i-k) \frac{(\lambda_i L)^j}{j!} e^{-\lambda_i L} . \tag{7}$$

By defining $\alpha_i(k) := \sum_{j=k}^{\infty} \frac{(\lambda_i L)^j}{j!} e^{-\lambda_i L}$, formula (7) can be rewritten as

$$ES_i = \sum_{k=0}^{\Delta_i} \Phi_i(k) \{ \lambda_i L \, \alpha_i(k+s_i) - (k+s_i) \, \alpha_i(k+s_i+1) \} . \tag{8}$$

Furthermore,

$$EQ_i = \sum_{k=0}^{\Delta_i} (\Delta_i - k) \, \Phi_i(k) . \tag{9}$$

Once the probability function $\Phi_i(k)$ of the residual stock has been calculated, $\mathcal{F}_i$ can be obtained from (6), (8) and (9).

***Algorithm to determine $s_i$ given the vector $\Delta$***

Step 1: Determine the probability function $\Phi_i(k)$, $k=0,..,\Delta_i$ from (a1) and (a3).

Step 2a: Initialize $s_i:=0$; calculate $EQ_i$ from (9).

Step 2b: Calculate $ES_i$ from (8).

Step 2c: Stop if $ES_i < (1-\beta)EQ_i$; otherwise increase $s_i$ by one unit and go back to Step 2b.

### *3.3. Solution method to determine parameters of the optimal (S,S-1,s) policy*

The results of Section 3.1 and 3.2 can be used to determine the optimal must-order points and the corresponding cost for a *given* vector $\Delta$. Now, an iterative solution method will be proposed to find an approximation for the vector $\Delta$ of the optimal (S,S-1,s) policy. The heuristic is outlined in the following algorithm.

***Algorithm to determine the optimal vector $\Delta$***

Step 1: Determine $T_D = \sqrt{\dfrac{2(A+\sum_{i=1}^{N} a_i)}{\sum_{i=1}^{N} \lambda_i h_i}}$;

For all items i, determine the integer value of $\Delta_i$ for which the

difference between $\sum_{j=0}^{\Delta_i} \dfrac{(\lambda_i T_D)^j}{j!} e^{-\lambda_i T_D}$ *and* $\dfrac{N}{N+1}$ is minimal;

Determine the corresponding must-order points by the method of Section 3.2 and calculate the cost C by formula (1); Set $C_{min}=C$.

Step 2: i:=0;

Repeat (until i=N)

- i:=i+1;
- Carry out an one dimensional search on $\Delta_i$ by the Golden-Section heuristic;
- Update $\Delta_i$ and $C_{min}$ if a better solution has been found.

Step 3: Stop if the vector $\Delta$ has not been changed in Step 2 or $C_{min}$ has not been decreased by more that $\epsilon$%; otherwise go back to Step 2.

The starting value for $\Delta$ (Step 1) has been suggested by Love (1979) in a related context (Love provides no motivation for this heuristic). Note that $T_D$ is the optimal length of an order cycle in the deterministic demand case. (From prior numerical examples it appeared that the obvious choice of $\Delta_i = \lambda_i T_D$ does not work satisfactorily).

In every iteration (Step 2), an one dimensional search is carried out for each item: $\Delta_i$ is varied, while the other $\Delta_j$, $j \neq i$, remain the same. To save computation time, the Golden-Section heuristic is used (which assumes convexity of C in $\Delta_i$). For every evaluation of a possible value of $\Delta_i$, the must order points of *all* items have to be calculated (since $\Delta_i$ can also affect other must-order points), together with the corresponding cost for the *whole* family. The iterative process terminates as soon as the vector $\Delta$ remains the same in two successive iterations or the minimal cost has been decreased less than a prespecified percentage of $\epsilon$%.

## 4. Numerical results

The above procedure has been applied on several numerical examples. Two families of items are considered, consisting of 4 and 8 items. The values of $\lambda_i$, $a_i$ and $h_i$ are listed in Table 2 for both families. For different experiments the lead time L is varied over two level (0.2 and 1), the required service level $\beta$ is also varied over two levels (0.95 and 0.99) and the major set-up cost A is varied over three levels (25,250,500). Detailed results of the 24 examples are given in Appendix 3.

**Table 2:** Data for numerical examples

| family with N=4 | | | | family with N=8 | | | |
|---|---|---|---|---|---|---|---|
| item i | $\lambda_i$ | $a_i$ | $h_i$ | item i | $\lambda_i$ | $a_i$ | $h_i$ |
| 1 | 20 | 10 | 5 | 1 | 20 | 10 | 5 |
| 2 | 15 | 20 | 5 | 2 | 15 | 10 | 5 |
| 3 | 10 | 30 | 5 | 3 | 10 | 20 | 5 |
| 4 | 5 | 40 | 5 | 4 | 5 | 20 | 5 |
| | | | | 5 | 20 | 30 | 5 |
| | | | | 6 | 15 | 30 | 5 |
| | | | | 7 | 10 | 40 | 5 |
| | | | | 8 | 5 | 40 | 5 |

The performance of a given coordinated replenishment strategy is measured by the percentage cost saving over the optimal independent (S,s) strategy. The optimal (S,s) strategy can be obtained by the approach of Federgruen et al. (1984) with $\mu_i$ =0 and $c_i = s_i$ for all items i. The cost which is computed from the model is exact because no assumption on the arrival process of the special replenishment opportunities is needed ($\mu_i = 0$). The percentage cost saving is calculated as

$$\% c.s. = 100 \cdot \frac{\textit{cost of independent strategy} - \textit{cost of coordinated strategy}}{\textit{cost of independent strategy}} . \quad (10)$$

First, the performance of the optimal (S,S-1,s) strategy is compared with the performance of the optimal (S,c,s) strategy, obtained by the traditional approach of Federgruen et al. Table 3 gives the average percentage cost saving for fixed values of the set-up cost ratio $A/\bar{a}$. Note that the performance of the (S,c,s) strategy is based on the real (simulated) cost of the strategy that follows from the model.

**Table 3:** Average % c.s. of optimal (S,S-1,s) and (S,c,s) policy

| $A/\bar{a}$ | (S,S-1,s) | (S,c,s) |
|---|---|---|
| 1 | 0.07 | 8.88 |
| 10 | 34.80 | 30.73 |
| 20 | 41.02 | 35.43 |

Note: the average performance for a fixed set-up cost ratio is based on 8 observations.

As expected, the (S,S-1,s) policy performs less than the (S,c,s) policy for the low set-up cost ratio. In some individual cases, the optimal (S,S-1,s) strategy has even a higher cost than the optimal (S,s) strategy. However, the (S,S-1,s) policy outperforms the (S,c,s) policy for high set-up cost ratios. It can be noted that the differences would even be larger if the cost from the *model* of Federgruen et al. had been used, as Atkins and Iyogun (1988) and Pantumsinchai (1992) do.

Atkins and Iyogun (1988) and Pantumsinchai (1992) conclude from their numerical experiments that the can-order policy may perform very poor, relative to QS and RS policies, for high set-up cost ratios. In these situations, we recommend to use the (S,S-1,s) policy, where the parameters are determined by the method in Section 3. The model of Federgruen et al. should be used for low set-up cost ratios. Let CAN be the best can-order strategy in a given situation. Based on numerical experience, we suggest the following rule of thumb to determine the parameters of CAN:

***Procedure to determine the parameters of CAN***

- If $A/\bar{a} \leq 2$ : Use the model of Federgruen et al. to determine the parameters $S_i$, $c_i$ and $s_i$ for each item i.
- If $2 < A/\bar{a} < 5$ : Determine the parameters $S_i$,$c_i$ and $s_i$ for each item i with the model of Federgruen et al. and with the model in Section 3; Choose the parameters according to the strategy with the lowest cost.
- If $A/\bar{a} \geq 5$ : Use the method of Section 3 to obtain the parameters $\Delta_i$ and $s_i$ for all items i; Set $S_i := s_i + \Delta_i$ and $c_i := S_i - 1$.

The performance of CAN is compared with the performance of the optimal QS and RS policy for our 24 examples. Atkins and Iyogun (1988) and Pantumsinchai (1992) give algorithms to calculate the optimal parameters for respectively a RS policy and a QS policy in case of stock-out costs. However, it is easy to adapt their algorithms to the service level case. (Atkins and Iyogun tried two versions of the periodic RS policy, called P and MP. Under P, the review period $R_i$ is equal for all items, whereas $R_i$ is an integer multiple $k_i$ (which can be different for several items) of some basic period under MP. Details are given in their paper. The adapted version of P has been used in our numerical analysis). The average performance of CAN and the optimal QS and RS strategy is shown for fixed values of $A/\bar{a}$, L, and $\beta$ in Table 4. To compare our results with the results of the other comparative studies, the performance according to the cost from the *model* of Federgruen et al. (FED) has also been calculated.

By comparing the performance of the RS and the QS policy on one side and FED on the other side, the same conclusions can be drawn as in earlier studies. However, if the performance of the RS and the QS policy is compared with CAN, then it appears that the can-order policy performs at least equally well as the other policies, even for high set-up cost ratios. This supports our conjecture that the poor performance of the can-order policy in some cases is due to the method to determine the control parameters and not to the policy itself.

It seems that the service level has a significant impact on the percentage cost saving of, in particular, the RS and the QS policy. (This was already noted by Pantumsinchai (1992) for the QS policy with respect to the stock-out cost). The percentage cost savings are higher for the family of 8 items. However, the relative performance of the different policies is not affected by the number of items.

**Table 4:** Average % c.s. of several policies

| factor | RS | QS | CAN | FED |
|---|---|---|---|---|
| $A/\bar{a}$ (8 observations) | | | | |
| 1 | -1.89 | 0.93 | 8.88 | 7.72 |
| 10 | 34.70 | 35.71 | 34.78 | 21.83 |
| 20 | 40.83 | 42.13 | 41.01 | 24.26 |
| L (12 observations) | | | | |
| 0.2 | 24.29 | 26.44 | 28.78 | 18.25 |
| 1.0 | 24.39 | 26.07 | 27.69 | 17.63 |
| $\beta$ (12 observations) | | | | |
| 0.95 | 28.41 | 29.92 | 30.19 | 19.34 |
| 0.99 | 20.27 | 22.58 | 26.28 | 16.53 |

This section will be closed with some remarks on the misspecification in the cost when the special replenishment opportunity model is used. The percentage cost error of the model of Federgruen et al. is defined by:

$$\%\,c.e. = 100 \cdot \frac{\textit{cost of model} - \textit{actual cost}}{\textit{actual cost}} . \tag{11}$$

It has already been mentioned that the percentage cost error will be very large for an arbitrary (S,S-1,s) strategy. This conjecture is verified by calculating the cost of the optimal (S,S-1,s) strategy (obtained with the approach in Section 3) with the method of Federgruen et al. Recall that $\mu_j = \sum_{i \neq j} \xi_i$, where $\xi_i$ denotes the expected number of replenishments per unit time that is triggered by item i. Note that $\xi_i$ is equal to $\Phi_i(0)/\tau$. The average percentage cost error of the (S,c,s) strategy, calculated by the approach of Federgruen et al., has also been calculated. Table 5 shows that the average percentage cost error is very large for high set-up cost ratios. The cost errors are dramatic for the (S,S-1,s) strategy. Hence, the model of Federgruen et al. will neglect such a policy, when searching for the optimal can-order policy.

**Table 5:** Average % c.e. of (S,c,s) and (S,S-1,s) strategy

| $A/\bar{a}$ | (S,c,s) | (S,S-1,s) |
|---|---|---|
| 1 | 1.23 | 5.75 |
| 10 | 11.85 | 30.18 |
| 20 | 14.91 | 37.73 |

## 5. Conclusions

Our analysis shows that can-order policies indeed do not outperform other coordinated replenishment policies like RS or QS policies. Nevertheless, the conclusions made in the comparative studies of Atkins and Iyogun (1988) and Pantumsinchai (1992) are wrong. It has been shown that the performance of the can-order policy ought not to be evaluated by the special replenishment opportunity model, suggested by Silver (1974) and Federgruen et al. (1984), in situations with high set-up cost ratios, because this model gives inaccurate results in such circumstances. For the case of Poisson demands, we developed a solution method to find the parameters of a (S,S-1,s) policy, which is a close to optimal can-order policy in situations with high set-up cost ratios. Numerical analysis points out that a properly chosen combination of both solution techniques leads to a can-order strategy which performs as well as the optimal RS or QS policy, as distinct from conclusions in the above mentioned comparative studies.

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**Appendix 1: Determination of probability function of residual stock**

It turns out that the problem of determining the probability function of the residual stock is an important issue. A similar problem was solved by Miltenburg and Silver (1984a,b) for the situation where the inventory position of each item is modelled as a diffusion process. For this situation, they showed that the probability distribution function has a specific form and developed some heuristics to estimate the shape and location parameters.

In the case of Poisson demand, define $T_i$, $f_i(t)$ and $F_i(t)$ as in Section 3. The probability that item i triggers the following order is equal to the probability that $T_i$ is smaller than all the other $T_j$. Hence,

$$\Phi_i(0) = Pr\{T_i < T_j, \forall j \neq i\} = \int_{t=0}^{\infty} f_i(t) \prod_{j \neq i} Pr\{T_j > t\}\, dt = \int_{t=0}^{\infty} f_i(t) \prod_{j \neq i} (1 - F_j(t))\, dt \,. \tag{a1}$$

Now, define $T^{(-i)}$ as the time until any item $j \neq i$ reaches its must-order point if item i is left out of consideration, i.e. $T^{(-i)} = \min_{j \neq i} T_j$, and denote the distribution function and the probability density function of $T^{(-i)}$ by $F^{(-i)}(t)$ and $f^{(-i)}(t)$.
So,

$$F^{(-i)}(t) = 1 - \prod_{j \neq i} (1 - F_j(t)) \,,$$

and (a2)

$$f^{(-i)}(t) = \sum_{j \neq i} f_j(t) \prod_{k \neq i,j} (1 - F_k(t)) \,.$$

*Lemma:*

The probability that the residual stock of item i is k ($k = 1,..,\Delta_i$) is equal to:

$$\Phi_i(k) = \int_{s=0}^{\infty} \frac{(\lambda_i s)^{(\Delta_i - k)}}{(\Delta_i - k)!} e^{-\lambda_i s} f^{(-i)}(s)\, ds, \quad k = 1,..,\Delta_i \,. \tag{a3}$$

*Proof:*

We present a formal proof of (a3) for the case $N=2$. This proof can be straightforwardly generalized to the case $N>2$, by replacing $T_2$ by $\min_{j\neq i} T_j$. Define,

$X_1(s)$: the excess stock above the must-order point of item 1 at time s.

Then, for $k>0$, $\Phi_1(k)=$

$$\int_{s=0}^{\infty} Pr(X_1(\min(T_1,T_2))=k) \mid T_1>s, T_2=s)\, d\,Pr(T_1>s, T_2=s) +$$

$$\int_{s=0}^{\infty} Pr(X_1(\min(T_1,T_2))=k) \mid T_1<s, T_2=s)\, d\,Pr(T_1<s, T_2=s)$$

$$= \int_{s=0}^{\infty} Pr(X_1(s)=k \mid T_1>s, T_2=s)\, d\,Pr(T_1>s, T_2=s)$$

$$= \int_{s=0}^{\infty} Pr(X_1(s)=k \mid T_1>s, T_2=s)\, Pr(T_1>s)\, d\,Pr(T_2=s)$$

$$= \int_{s=0}^{\infty} Pr(X_1(s)=k, T_1>s \mid T_2=s)\, d\,Pr(T_2=s)$$

$$= \int_{s=0}^{\infty} Pr(X_1(s)=k \mid T_2=s)\, d\,Pr(T_2=s)$$

$$= \int_{s=0}^{\infty} \frac{(\lambda_1 s)^{(\Delta_1-k)}}{(\Delta_1-k)} e^{-\lambda_1 s} f^{(-1)}(s)\, ds$$

Numerical integration can be used to approximate the probability function of the residual stock from (a1) and (a3).

Another expression for the probabilty function of the residual stock can be obtained as follows. Let $\gamma_i = \lambda_i/(\Sigma_j \lambda_j)$. Then, $\Phi_i(0)$ can be calculated from

$$\gamma_i^{\Delta_i} \sum_{j_1=0}^{\Delta_1-1} \gamma_1^{j_1} \cdots \sum_{j_{i-1}=0}^{\Delta_{i-1}-1} \gamma_{i-1}^{j_{i-1}} \sum_{j_{i+1}=0}^{\Delta_{i+1}-1} \gamma_{i+1}^{j_{i+1}} \cdots \sum_{j_N=0}^{\Delta_N-1} \gamma_N^{j_N} C_i(j_1,..,j_N) ,$$

where,

$$C_i(j_1,..,j_N) = \frac{(\Delta_i - 1 + \sum_{v \neq i} j_v)!}{(\Delta_i - 1)! \prod_{v \neq i} (j_v!)} .$$

It can also be shown that $\Phi_i(b)$, for $b=1,..,\Delta_i$, is equal to

$$\gamma_i^{\Delta_i - b} \sum_{k \neq i} \gamma_k^{\Delta_k} \sum_{j_1=0}^{\Delta_1-1} \gamma_1^{j_1} \cdots \sum_{j_{i-1}=0}^{\Delta_{i-1}-1} \gamma_{i-1}^{j_{i-1}} \sum_{j_{i+1}=0}^{\Delta_{i+1}-1} \gamma_{i+1}^{j_{i+1}} \cdots \sum_{j_{k-1}=0}^{\Delta_{k-1}-1} \gamma_{k-1}^{j_{k-1}} \sum_{j_{k+1}=0}^{\Delta_{k+1}-1} \gamma_{k+1}^{j_{k+1}} \cdots \sum_{j_N=0}^{\Delta_N-1} \gamma_N^{j_N} C_{i,k}(j_1,..,j_N)$$

where,

$$C_{i,k}(j_1,..,j_N) = \frac{(\Delta_i - b + \Delta_k - 1 + \sum_{v \neq i,k} j_v)!}{(\Delta_i - b)!(\Delta_k - 1)! \prod_{v \neq i,k} (j_v!)} .$$

It is obvious that this expression is numerically intractable when N or $\Delta_i$ (for some i) is large.

**Appendix 2: Determination of $H_i(x,y,t)$**

Recall that $H_i(x,y,t)$ is expected holding cost for item i during an order cycle of t periods given that the inventory on hand equals x at the beginning and equals y at the end of the cycle. It can be shown that the (x-y) demands are homogeneously distributed over [0,t] (see e.g. Tijms (1986)). Five different situations are distinguished, depending on whether x and y are positive or negative and whether the particular item triggers the order or not. Note that in case $x-y=\Delta_i$ the last demand of item i was at time t (since the item triggers the order). The following formula for $H_i(x,y,t)$ summarizes all five different cases:

$$H_i(x,y,t) = \begin{cases} h_i \frac{t}{2}(x+y) & \text{if } x>0, y\geq 0, x-y<\Delta_i, \\ h_i \frac{t}{2}(x+y+1) & \text{if } x>0, y\geq 0, x-y=\Delta_i, \\ h_i \frac{t}{2}\frac{x(x+1)}{(x-y+1)} & \text{if } x>0, y<0, x-y<\Delta_i, \\ h_i \frac{t}{2}\frac{x(x+1)}{(x-y)} & \text{if } x>0, y<0, x-y=\Delta_i, \\ 0 & \text{if } x\leq 0 . \end{cases} \tag{a4}$$

**Appendix 3: Numerical results**

Before a detailed list of the output of the 24 examples is given, some remarks are made on the numerical integration method which is used to solve the integral equations (3), (5), (a1) and (a3). Note that these equations have the following form: $\int_{t=0}^{\infty} \mathcal{L}(t)\, dt$,

where $\mathcal{L}(t)$ is a complex function of t which can not be simplified.

***Algorithm for numerical integration***

Step 1: Determine $t_{max}$ such that the cumulative probability density of t higher than $t_{max}$ is less than $10^{-3}$.

Step 2: Determine 25 integration points, where $u_1 := 0$ and $u_j := u_{j-1} + t_{max}/24$ for $j = 2,..,25$.

Step 3: The function $\mathcal{L}(t)$ is approximated by a piece-wise linear function

$$\hat{\mathcal{L}}(t) = a_j t + b_j \quad for\ t \in [u_j, u_{j+1}] \quad j = 1,..,24\ ,$$

where $a_j$ and $b_j$ are given by

$$a_j = \frac{\mathcal{L}(u_{j+1}) - \mathcal{L}(u_j)}{u_{j+1} - u_j} \quad , \quad b_j = \frac{\mathcal{L}(u_j)u_{j+1} - \mathcal{L}(u_{j+1})u_j}{u_{j+1} - u_j}\ .$$

Calculate $a_j$ and $b_j$ for $j = 1,..,24$.

Step 4: Calculate

$$\int_{t=0}^{\infty} \mathcal{L}(t)\,dt \approx \sum_{j=1}^{24} \int_{t=u_j}^{u_{j+1}} \hat{\mathcal{L}}(t)\,dt = \sum_{j=1}^{24} \int_{t=u_j}^{u_{j+1}} \{a_j t + b_j\}\,dt =$$

$$\sum_{j=1}^{24} \left\{ \frac{1}{2} a_j (u_{j+1}^2 - u_j^2) + b_j (u_{j+1} - u_j) \right\}.$$

The cost C, calculated by formula (1) deviated at most 1% from the simulated cost for the optimal (S,S-1,s) strategy in our 24 examples. $\epsilon$ was set equal to 0.1% in the experiments. The number of iterations (including the determination of the starting values) varied between 2 and 5.

The detailed results for each example are presented in Table A.1. De input-parameters N, L, $\beta$ and A are already defined. The variables C1 up to C7 are explained below the table.

**Table A.1:** Detailed numerical results

| ex. | N | L | $\beta$ | A | C1 | C2 | C3 | C4 | C5 | C6 | C7 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 0.2 | 0.95 | 25 | 279.7 | 275.2 | 321.7 | 299.1 | 307.4 | 294.4 | 308.8 |
| 2 | 4 | 0.2 | 0.95 | 250 | 563.2 | 493.0 | 626.5 | 469.0 | 467.9 | 456.7 | 693.7 |
| 3 | 4 | 0.2 | 0.95 | 500 | 766.2 | 655.2 | 864.2 | 608.3 | 590.3 | 577.5 | 955.0 |
| 4 | 4 | 0.2 | 0.99 | 25 | 316.1 | 311.5 | 365.4 | 342.7 | 370.3 | 357.4 | 333.4 |
| 5 | 4 | 0.2 | 0.99 | 250 | 606.4 | 541.9 | 670.5 | 520.0 | 556.1 | 531.2 | 737.5 |
| 6 | 4 | 0.2 | 0.99 | 500 | 820.9 | 713.1 | 923.6 | 658.5 | 693.6 | 667.7 | 1003.7 |
| 7 | 4 | 1.0 | 0.95 | 25 | 314.5 | 311.6 | 349.5 | 335.3 | 333.0 | 326.1 | 341.8 |
| 8 | 4 | 1.0 | 0.95 | 250 | 579.5 | 521.5 | 638.0 | 484.1 | 488.7 | 477.2 | 711.9 |
| 9 | 4 | 1.0 | 0.95 | 500 | 761.4 | 666.9 | 799.7 | 606.0 | 608.0 | 598.9 | 972.6 |
| 10 | 4 | 1.0 | 0.99 | 25 | 374.4 | 371.0 | 418.4 | 400.3 | 420.0 | 402.5 | 385.8 |
| 11 | 4 | 1.0 | 0.99 | 250 | 659.5 | 598.7 | 719.2 | 569.0 | 588.7 | 571.4 | 788.6 |
| 12 | 4 | 1.0 | 0.99 | 500 | 847.8 | 747.5 | 957.2 | 700.5 | 721.8 | 700.7 | 1044.4 |
| 13 | 8 | 0.2 | 0.95 | 25 | 547.9 | 538.4 | 0.00 | 599.5 | 592.3 | 576.8 | 622.0 |
| 14 | 8 | 0.2 | 0.95 | 250 | 1007.2 | 866.6 | 0.00 | 812.3 | 771.7 | 764.6 | 1391.3 |
| 15 | 8 | 0.2 | 0.95 | 500 | 1356.7 | 1111.3 | 0.00 | 1004.6 | 935.3 | 920.3 | 1912.9 |
| 16 | 8 | 0.2 | 0.99 | 25 | 625.1 | 616.1 | 0.00 | 696.5 | 711.6 | 697.7 | 671.5 |
| 17 | 8 | 0.2 | 0.99 | 250 | 1103.9 | 965.9 | 0.00 | 913.5 | 935.3 | 911.2 | 1479.1 |
| 18 | 8 | 0.2 | 0.99 | 500 | 1429.2 | 1186.2 | 0.00 | 1078.8 | 1112.3 | 1085.3 | 2010.5 |
| 19 | 8 | 1.0 | 0.95 | 25 | 617.9 | 611.4 | 0.00 | 652.4 | 643.3 | 636.1 | 685.8 |
| 20 | 8 | 1.0 | 0.95 | 250 | 1049.1 | 924.9 | 0.00 | 830.7 | 827.4 | 814.6 | 1427.8 |
| 21 | 8 | 1.0 | 0.95 | 500 | 1344.6 | 1117.8 | 0.00 | 992.0 | 977.5 | 965.4 | 1943.6 |
| 22 | 8 | 1.0 | 0.99 | 25 | 730.1 | 722.9 | 0.00 | 810.7 | 814.7 | 796.8 | 789.9 |
| 23 | 8 | 1.0 | 0.99 | 250 | 1197.7 | 1066.3 | 0.00 | 1018.8 | 1011.7 | 993.8 | 1566.8 |
| 24 | 8 | 1.0 | 0.99 | 500 | 1532.5 | 1299.8 | 0.00 | 1170.1 | 1177.5 | 1159.9 | 2092.0 |

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Legend to Table A.1.

C1 : cost calculated from the model of Federgruen et al. for the (S,c,s) strategy obtained by the same model;

C2 : simulated cost for the (S,c,s) strategy obtained by the model of Federgruen et al.;

C3 : cost calculated from the model of Federgruen et al. for the (S,S-1,s) strategy obtained by the algorithm in Section 3;

C4 : exact cost calculated by formula (1) for the (S,S-1,s) strategy obtained by the algorithm in Section 3;

C5 : exact cost according to optimal RS policy obtained by an adapted version of the method of Atkins and Iyogun (1988) (the optimal R was found by a grid search with steps of 0.05);

C6 : exact cost according to the optimal QS policy obtained by an adapted version of the method of Pantumsinchai (1992);

C7 : exact cost according to the optimal (S,s) policy obtained by the model of Federgruen et al. (1984).

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